3.5 Graphing Linear Equations in Slope-Intercept Form

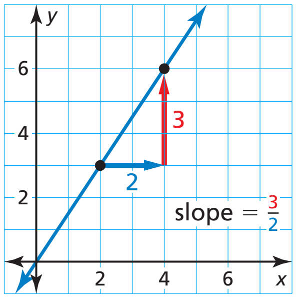
Essential Question

How can you describe the graph of the equation $y = mx + b$?

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

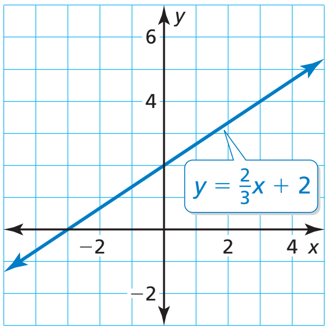
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



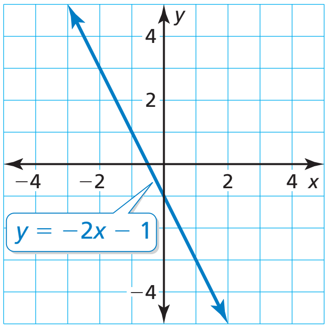
EXPLORATION 1 Finding Slopes and y-Intercepts

Work with a partner. Find the slope and y-intercept of each line.

a.



b.



MAKING CONJECTURES

To be proficient in math, you first need to collect and organize data. Then make conjectures about the patterns you observe in the data.

EXPLORATION 2 Writing a Conjecture

Work with a partner. Graph each equation. Then copy and complete the table. Use the completed table to write a conjecture about the relationship between the graph of $y = mx + b$ and the values of m and b.

Equation	Description of graph	Slope of graph	y-Intercept
a. $y = -\frac{2}{3}x + 3$	Line	$-\frac{2}{3}$	3
b. $y = 2x - 2$			
c. $y = -x + 1$			
d. $y = x - 4$			

Communicate Your Answer

3. How can you describe the graph of the equation $y = mx + b$?

a. How does the value of m affect the graph of the equation?

b. How does the value of b affect the graph of the equation?

c. Check your answers to parts (a) and (b) by choosing one equation from Exploration 2 and (1) varying only m and (2) varying only b.

3.5 Lesson

Core Vocabulary

slope, *p. 136*
rise, *p. 136*
run, *p. 136*
slope-intercept form, *p. 138*
constant function, *p. 138*

Previous
dependent variable
independent variable

What You Will Learn

- Find the slope of a line.
- Use the slope-intercept form of a linear equation.
- Use slopes and *y*-intercepts to solve real-life problems.

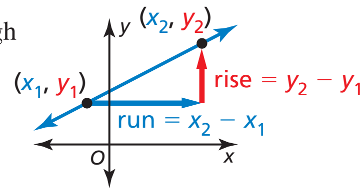
The Slope of a Line

Core Concept

Slope

The **slope** m of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2) is the ratio of the **rise** (change in y) to the **run** (change in x).

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

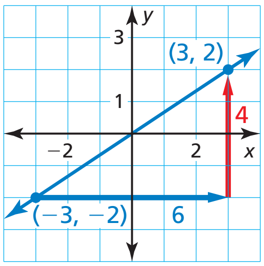


When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

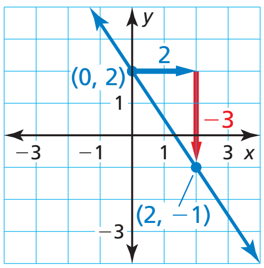
EXAMPLE 1 Finding the Slope of a Line

Describe the slope of each line. Then find the slope.

a.



b.



STUDY TIP

When finding slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2). The result is the same.

SOLUTION

a. The line rises from left to right. So, the slope is positive. Let $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

b. The line falls from left to right. So, the slope is negative. Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{2 - 0} = \frac{-3}{2} = -\frac{3}{2}$$

READING

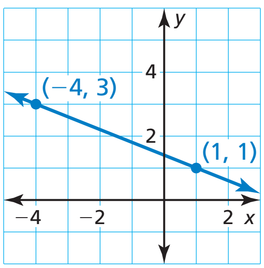
In the slope formula, x_1 is read as "*x* sub one" and y_2 is read as "*y* sub two." The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

Monitoring Progress

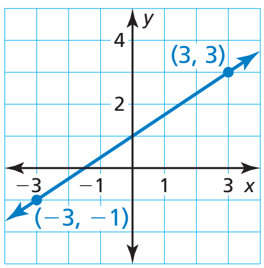
Help in English and Spanish at *BigIdeasMath.com*

Describe the slope of the line. Then find the slope.

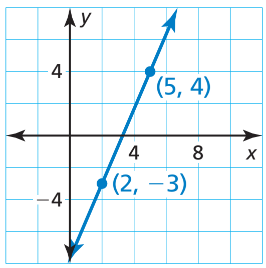
1.



2.



3.



EXAMPLE 2 Finding Slope from a Table

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

a.

x	y
4	20
7	14
10	8
13	2

b.

x	y
-1	2
1	2
3	2
5	2

c.

x	y
-3	-3
-3	0
-3	6
-3	9

STUDY TIP

As a check, you can plot the points represented by the table to verify that the line through them has a slope of -2.

SOLUTION

a. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (4, 20)$ and $(x_2, y_2) = (7, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 20}{7 - 4} = \frac{-6}{3}, \text{ or } -2$$

▶ The slope is -2.

b. Note that there is no change in y. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (5, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-1)} = \frac{0}{6}, \text{ or } 0$$

The change in y is 0.

▶ The slope is 0.

c. Note that there is no change in x. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (-3, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{-3 - (-3)} = \frac{6}{0} \;✗$$

The change in x is 0.

▶ Because division by zero is undefined, the slope of the line is undefined.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

4.

x	2	4	6	8
y	10	15	20	25

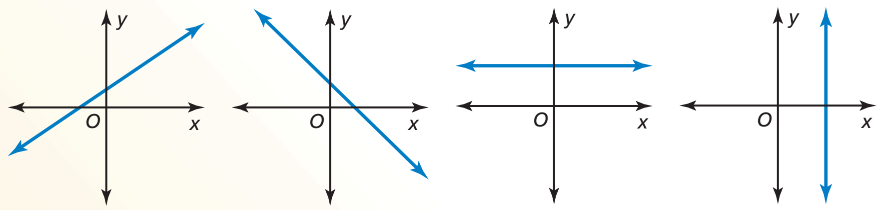
5.

x	5	5	5	5
y	-12	-9	-6	-3

Concept Summary

Slope

Positive slope | *Negative slope* | *Slope of 0* | *Undefined slope*



The line rises from left to right. | The line falls from left to right. | The line is horizontal. | The line is vertical.

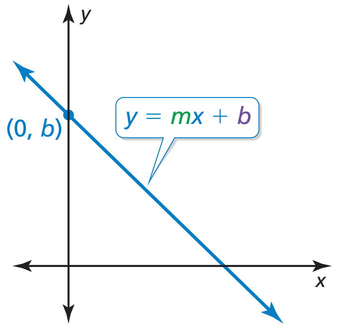
Using the Slope-Intercept Form of a Linear Equation

Core Concept

Slope-Intercept Form

Words A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.

Algebra $y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$



A linear equation written in the form $y = 0x + b$, or $y = b$, is a **constant function**. The graph of a constant function is a horizontal line.

EXAMPLE 3 Identifying Slopes and y-Intercepts

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = 3x - 4$ **b.** $y = 6.5$ **c.** $-5x - y = -2$

SOLUTION

a. $y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$ Write the slope-intercept form.

$y = 3x + (-4)$ Rewrite the original equation in slope-intercept form.

▶ The slope is 3, and the y-intercept is -4.

STUDY TIP

For a constant function, every input has the same output. For instance, in Example 3b, every input has an output of 6.5.

b. The equation represents a constant function. The equation can also be written as $y = 0x + 6.5$.

▶ The slope is 0, and the y-intercept is 6.5.

STUDY TIP

When you rewrite a linear equation in slope-intercept form, you are expressing y as a function of x.

c. Rewrite the equation in slope-intercept form by solving for y.

$$-5x - y = -2 \quad \text{Write the original equation.}$$

$$\underline{+5x} \qquad \underline{+5x} \quad \text{Add } 5x \text{ to each side.}$$

$$-y = 5x - 2 \quad \text{Simplify.}$$

$$\frac{-y}{-1} = \frac{5x - 2}{-1} \quad \text{Divide each side by } -1.$$

$$y = -5x + 2 \quad \text{Simplify.}$$

▶ The slope is -5, and the y-intercept is 2.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the slope and the y-intercept of the graph of the linear equation.

6. $y = -6x + 1$ **7.** $y = 8$ **8.** $x + 4y = -10$

EXAMPLE 4 Using Slope-Intercept Form to Graph

Graph $2x + y = 2$. Identify the x-intercept.

SOLUTION

Step 1 Rewrite the equation in slope-intercept form.

$$y = -2x + 2$$

Step 2 Find the slope and the y-intercept.

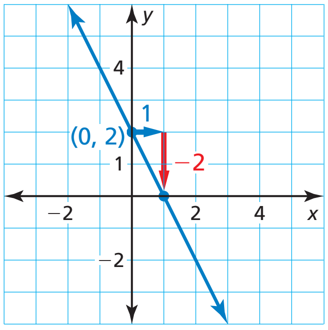
$$m = -2 \text{ and } b = 2$$

Step 3 The y-intercept is 2. So, plot (0, 2).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

Plot the point that is 1 unit right and 2 units down from (0, 2). Draw a line through the two points.



▶ The line crosses the x-axis at (1, 0). So, the x-intercept is 1.

STUDY TIP

You can use the slope to find points on a line in either direction. In Example 4, note that the slope can be written as $\frac{2}{-1}$. So, you could move 1 unit left and 2 units up from (0, 2) to find the point (–1, 4).

REMEMBER

You can also find the x-intercept by substituting 0 for y in the equation $2x + y = 2$ and solving for x.

EXAMPLE 5 Graphing from a Verbal Description

A linear function g models a relationship in which the dependent variable increases 3 units for every 1 unit the independent variable increases. Graph g when $g(0) = 3$. Identify the slope, y-intercept, and x-intercept of the graph.

SOLUTION

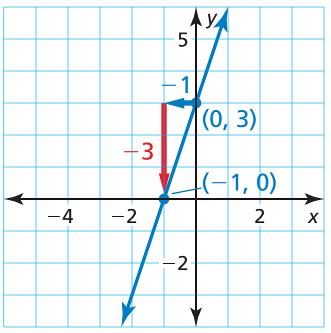
Because the function g is linear, it has a constant rate of change. Let x represent the independent variable and y represent the dependent variable.

Step 1 Find the slope. When the dependent variable increases by 3, the change in y is $+3$. When the independent variable increases by 1, the change in x is $+1$.

So, the slope is $\frac{3}{1}$, or 3.

Step 2 Find the y-intercept. The statement $g(0) = 3$ indicates that when $x = 0$, $y = 3$. So, the y-intercept is 3. Plot (0, 3).

Step 3 Use the slope to find another point on the line. A slope of 3 can be written as $\frac{-3}{-1}$. Plot the point that is 1 unit left and 3 units down from (0, 3). Draw a line through the two points. The line crosses the x-axis at $(-1, 0)$. So, the x-intercept is -1.



▶ The slope is 3, the y-intercept is 3, and the x-intercept is -1.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Graph the linear equation. Identify the x-intercept.

9. $y = 4x - 4$ **10.** $3x + y = -3$ **11.** $x + 2y = 6$

12. A linear function h models a relationship in which the dependent variable decreases 2 units for every 5 units the independent variable increases. Graph h when $h(0) = 4$. Identify the slope, y-intercept, and x-intercept of the graph.

Solving Real-Life Problems

In most real-life problems, slope is interpreted as a rate, such as miles per hour, dollars per hour, or people per year.

EXAMPLE 6 Modeling with Mathematics

A submersible that is exploring the ocean floor begins to ascend to the surface. The elevation h (in feet) of the submersible is modeled by the function $h(t) = 650t - 13{,}000$, where t is the time (in minutes) since the submersible began to ascend.

a. Graph the function and identify its domain and range.

b. Interpret the slope and the intercepts of the graph.

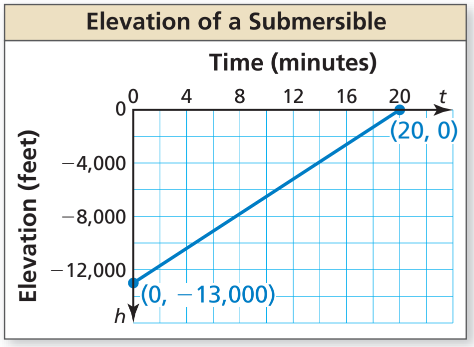
SOLUTION

1. **Understand the Problem** You know the function that models the elevation. You are asked to graph the function and identify its domain and range. Then you are asked to interpret the slope and intercepts of the graph.

2. **Make a Plan** Use the slope-intercept form of a linear equation to graph the function. Only graph values that make sense in the context of the problem. Examine the graph to interpret the slope and the intercepts.

3. **Solve the Problem**

   a. The time t must be greater than or equal to 0. The elevation h is below sea level and must be less than or equal to 0. Use the slope of 650 and the h-intercept of $-13{,}000$ to graph the function in Quadrant IV.



▶ The domain is $0 \le t \le 20$, and the range is $-13{,}000 \le h \le 0$.

   b. The slope is 650. So, the submersible ascends at a rate of 650 feet per minute. The h-intercept is $-13{,}000$. So, the elevation of the submersible after 0 minutes, or when the ascent begins, is $-13{,}000$ feet. The t-intercept is 20. So, the submersible takes 20 minutes to reach an elevation of 0 feet, or sea level.

4. **Look Back** You can check that your graph is correct by substituting the t-intercept for t in the function. If $h = 0$ when $t = 20$, the graph is correct.

$h = 650(20) - 13{,}000$ — Substitute 20 for t in the original equation.

$h = 0$ ✓ — Simplify.

STUDY TIP

Because t is the independent variable, the horizontal axis is the t-axis and the graph will have a "t-intercept." Similarly, the vertical axis is the h-axis and the graph will have an "h-intercept."

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

13. **WHAT IF?** The elevation of the submersible is modeled by $h(t) = 500t - 10{,}000$. (a) Graph the function and identify its domain and range. (b) Interpret the slope and the intercepts of the graph.

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The ________ of a nonvertical line passing through two points is the ratio of the rise to the run.

2. **VOCABULARY** What is a constant function? What is the slope of a constant function?

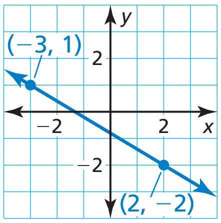
3. **WRITING** What is the slope-intercept form of a linear equation? Explain why this form is called the slope-intercept form.

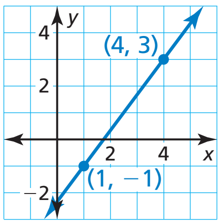
4. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

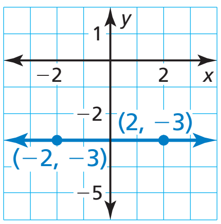
$y = -5x - 1$ $\quad$ $2x - y = 8$ $\quad$ $y = x + 4$ $\quad$ $y = -3x + 13$

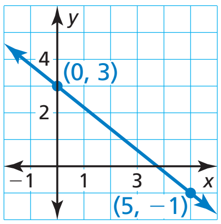
Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, describe the slope of the line. Then find the slope. *(See Example 1.)*

5. 

6. 

7. 

8. 

In Exercises 9–12, the points represented by the table lie on a line. Find the slope of the line. *(See Example 2.)*

9.

x	−9	−5	−1	3
y	−2	0	2	4

10.

x	−1	2	5	8
y	−6	−6	−6	−6

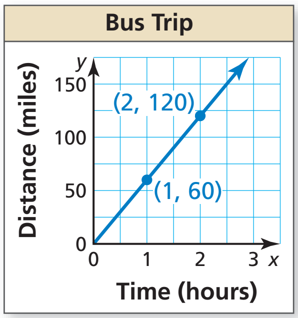
11.

x	0	0	0	0
y	−4	0	4	8

12.

x	−4	−3	−2	−1
y	2	−5	−12	−19

13. **ANALYZING A GRAPH** The graph shows the distance y (in miles) that a bus travels in x hours. Find and interpret the slope of the line.



14. **ANALYZING A TABLE** The table shows the amount x (in hours) of time you spend at a theme park and the admission fee y (in dollars) to the park. The points represented by the table lie on a line. Find and interpret the slope of the line.

Time (hours), *x*	Admission (dollars), *y*
6	54.99
7	54.99
8	54.99

In Exercises 15–22, find the slope and the y-intercept of the graph of the linear equation. *(See Example 3.)*

15. $y = -3x + 2$

16. $y = 4x - 7$

17. $y = 6x$

18. $y = -1$

19. $-2x + y = 4$

20. $x + y = -6$

21. $-5x = 8 - y$

22. $0 = 1 - 2y + 14x$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in finding the slope and the y-intercept of the graph of the equation.**

23.

$x = -4y$

The slope is -4, and the y-intercept is 0.

24.

$y = 3x - 6$

The slope is 3, and the y-intercept is 6.

In Exercises 25–32, graph the linear equation. Identify the x-intercept. *(See Example 4.)*

25. $y = -x + 7$

26. $y = \frac{1}{2}x + 3$

27. $y = 2x$

28. $y = -x$

29. $3x + y = -1$

30. $x + 4y = 8$

31. $-y + 5x = 0$

32. $2x - y + 6 = 0$

In Exercises 33 and 34, graph the function with the given description. Identify the slope, y-intercept, and x-intercept of the graph. *(See Example 5.)*

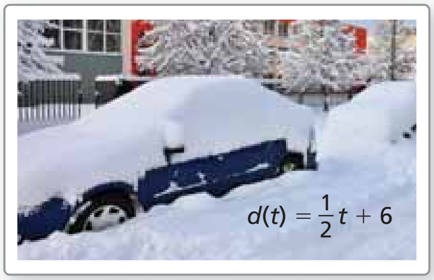
33. A linear function f models a relationship in which the dependent variable decreases 4 units for every 2 units the independent variable increases. The value of the function at 0 is -2.

34. A linear function h models a relationship in which the dependent variable increases 1 unit for every 5 units the independent variable decreases. The value of the function at 0 is 3.

35. **GRAPHING FROM A VERBAL DESCRIPTION** A linear function r models the growth of your right index fingernail. The length of the fingernail increases 0.7 millimeter every week. Graph r when $r(0) = 12$. Identify the slope and interpret the y-intercept of the graph.

36. **GRAPHING FROM A VERBAL DESCRIPTION** A linear function m models the amount of milk sold by a farm per month. The amount decreases 500 gallons for every \$1 increase in price. Graph m when $m(0) = 3000$. Identify the slope and interpret the x- and y-intercepts of the graph.

37. **MODELING WITH MATHEMATICS** The function shown models the depth d (in inches) of snow on the ground during the first 9 hours of a snowstorm, where t is the time (in hours) after the snowstorm begins. *(See Example 6.)*



a. Graph the function and identify its domain and range.

b. Interpret the slope and the d-intercept of the graph.

38. **MODELING WITH MATHEMATICS** The function $c(x) = 0.5x + 70$ represents the cost c (in dollars) of renting a truck from a moving company, where x is the number of miles you drive the truck.

a. Graph the function and identify its domain and range.

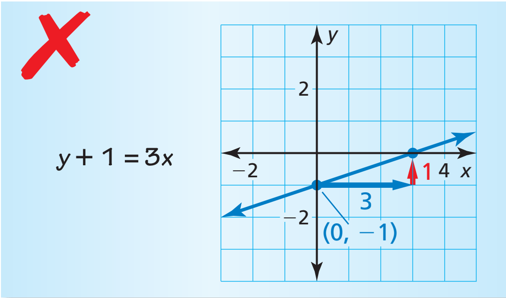
b. Interpret the slope and the c-intercept of the graph.

39. **COMPARING FUNCTIONS** A linear function models the cost of renting a truck from a moving company. The table shows the cost y (in dollars) when you drive the truck x miles. Graph the function and compare the slope and the y-intercept of the graph with the slope and the c-intercept of the graph in Exercise 38.

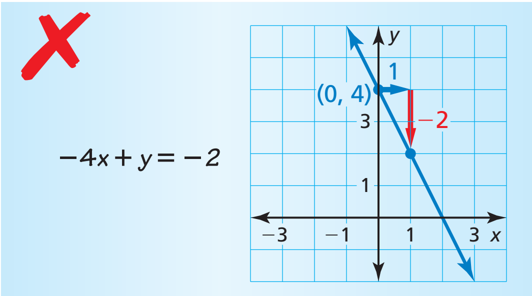
Miles, x	Cost (dollars), y
0	40
50	80
100	120

ERROR ANALYSIS **In Exercises 40 and 41, describe and correct the error in graphing the function.**

40.



41.



42. **MATHEMATICAL CONNECTIONS** Graph the four equations in the same coordinate plane.

$$3y = -x - 3$$

$$2y - 14 = 4x$$

$$4x - 3 - y = 0$$

$$x - 12 = -3y$$

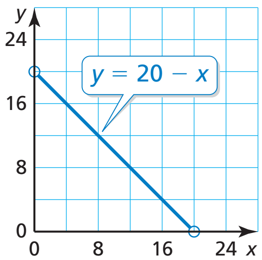
a. What enclosed shape do you think the lines form? Explain.

b. Write a conjecture about the equations of parallel lines.

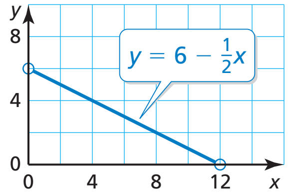
43. **MATHEMATICAL CONNECTIONS** The graph shows the relationship between the width y and the length x of a rectangle in inches. The perimeter of a second rectangle is 10 inches less than the perimeter of the first rectangle.

a. Graph the relationship between the width and length of the second rectangle.

b. How does the graph in part (a) compare to the the graph shown?



44. **MATHEMATICAL CONNECTIONS** The graph shows the relationship between the base length x and the side length (of the two equal sides) y of an isosceles triangle in meters. The perimeter of a second isosceles triangle is 8 meters more than the perimeter of the first triangle.

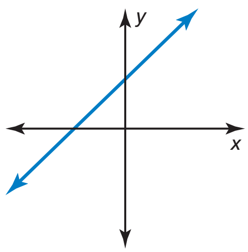


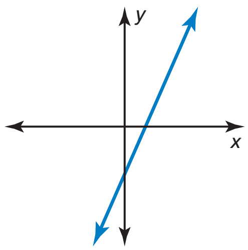
a. Graph the relationship between the base length and the side length of the second triangle.

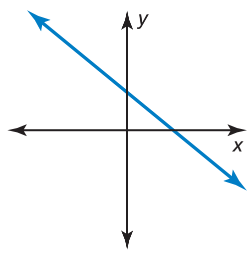
b. How does the graph in part (a) compare to the graph shown?

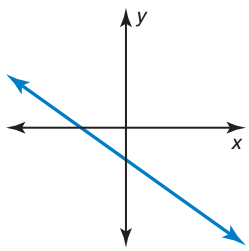
45. **ANALYZING EQUATIONS** Determine which of the equations could be represented by each graph.

$y = -3x + 8$	$y = -x - \frac{4}{3}$
$y = -7x$	$y = 2x - 4$
$y = \frac{7}{4}x - \frac{1}{4}$	$y = \frac{1}{3}x + 5$
$y = -4x - 9$	$y = 6$

a.


b.


c.


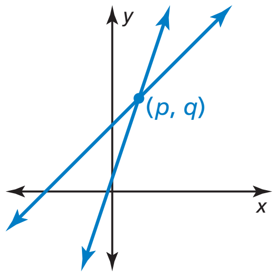
d.


46. **MAKING AN ARGUMENT** Your friend says that you can write the equation of any line in slope-intercept form. Is your friend correct? Explain your reasoning.

47. **WRITING** Write the definition of the slope of a line in two different ways.

48. **THOUGHT PROVOKING** Your family goes on vacation to a beach 300 miles from your house. You reach your destination 6 hours after departing. Draw a graph that describes your trip. Explain what each part of your graph represents.

49. **ANALYZING A GRAPH** The graphs of the functions $g(x) = 6x + a$ and $h(x) = 2x + b$, where a and b are constants, are shown. They intersect at the point (p, q).

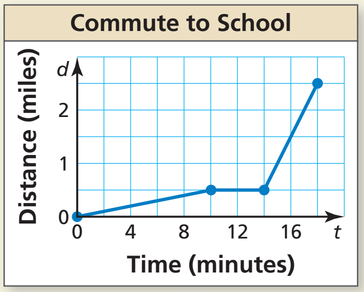


a. Label the graphs of g and h.

b. What do a and b represent?

c. Starting at the point (p, q), trace the graph of g until you get to the point with the x-coordinate $p + 2$. Mark this point C. Do the same with the graph of h. Mark this point D. How much greater is the y-coordinate of point C than the y-coordinate of point D?

50. **HOW DO YOU SEE IT?** You commute to school by walking and by riding a bus. The graph represents your commute.



a. Describe your commute in words.

b. Calculate and interpret the slopes of the different parts of the graph.

PROBLEM SOLVING **In Exercises 51 and 52, find the value of k so that the graph of the equation has the given slope or y-intercept.**

51. $y = 4kx - 5;\ m = \frac{1}{2}$

52. $y = -\frac{1}{3}x + \frac{5}{6}k;\ b = -10$

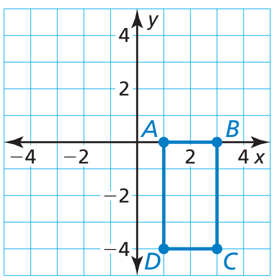
53. **ABSTRACT REASONING** To show that the slope of a line is constant, let (x_1, y_1) and (x_2, y_2) be any two points on the line $y = mx + b$. Use the equation of the line to express y_1 in terms of x_1 and y_2 in terms of x_2. Then use the slope formula to show that the slope between the points is m.

Maintaining Mathematical Proficiency

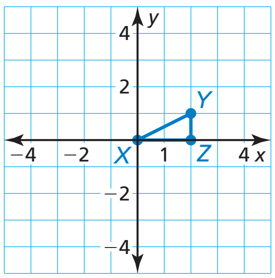
Reviewing what you learned in previous grades and lessons

Find the coordinates of the figure after the transformation. *(Skills Review Handbook)*

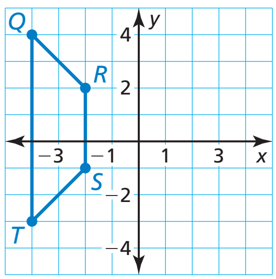
54. Translate the rectangle 4 units left.



55. Dilate the triangle with respect to the origin using a scale factor of 2.



56. Reflect the trapezoid in the y-axis.



Determine whether the equation represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

57. $y - 9 = \frac{2}{x}$

58. $x = 3 + 15y$

59. $\frac{x}{4} + \frac{y}{12} = 1$

60. $y = 3x^4 - 6$